



# The effect of mesoscales on the tracer equation in $z$ -coordinates OGCMs

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## Abstract

In  $z$ -coordinates ocean codes, mesoscale fluxes entering the T,S equations are represented by three terms: an eddy-induced velocity, a diapycnal flux  $\Sigma$  and a diffusion (Redi-like) term. Several eddy resolving codes have shown that the diapycnal flux  $\Sigma$  is quite large. However, all ocean codes have been run with zero diapycnal flux,  $\Sigma = 0$ .

We model  $\Sigma$  and show that its contribution is of the same order of magnitude as the other two mesoscale terms usually accounted for.

We also assess the validity of the two arguments most frequently cited to neglect  $\Sigma$ : (1) in an adiabatic regime, fluxes across isopycnal surfaces must vanish and so must the diapycnal flux  $\Sigma$  (we show that since  $\Sigma$  is not the total buoyancy flux but only part of it, there is no justification in demanding that  $\Sigma$  should satisfy the same conditions as the total flux) and (2) the results of a  $z$ -coordinate ocean code without  $\Sigma$  can be re-interpreted as those derived from the TRM (temporal residual mean) in which there is no  $\Sigma$  almost by definition since TRM is quite close to an isopycnal model.

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## 1. The problem

In  $z$ -coordinates, the general equation for an arbitrary mean tracer  $\bar{\tau}$  is given by

$$\bar{\tau}_t + \bar{\mathbf{U}} \cdot \nabla \bar{\tau} = -\nabla \cdot (\bar{\mathbf{U}'\tau'}) + Q \quad (1)$$

Here,  $\bar{\mathbf{U}}$  is the 3D mean velocity field,  $\bar{\mathbf{U}'\tau'}$  is the mesoscale-induced flux,  $\mathbf{U}' = (\mathbf{u}', w')$  is the mesoscale velocity field,  $\nabla = (\nabla_H, \partial_z)$ , an overbar indicates a fixed depth averaging and  $Q$  stands for irreversible (diabatic) terms. Coarse-resolution OGCMs are unable to resolve the mesoscale field and must model the 3D flux  $\bar{\mathbf{U}'\tau'}$ . In the adiabatic region of the ocean, mesoscales can be modeled with an *advective velocity* (called eddy-induced

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velocity, Gent and McWilliams, 1990; Canuto and Dubovikov, 2005, 2006), a *diffusive term* (Redi, 1982) and the  $z$ -derivative of a *diapycnal flux*  $\Sigma$ . Thus, Eq. (1) becomes:

$$\bar{\tau}_t + (\bar{\mathbf{U}} + \mathbf{U}^+) \cdot \nabla \bar{\tau} + \frac{\partial \Sigma}{\partial z} = D(\bar{\tau}) + Q \quad (2)$$

Here,  $\mathbf{U}^+ = (\mathbf{u}^+, w^+)$  is the 3D eddy-induced velocity,  $\Sigma$  is the mesoscale-induced diapycnal flux and  $D(\bar{\tau})$  is the diffusion term. In the case of buoyancy  $\tau = b = -g\rho_0^{-1}(\rho - \rho_0)$ , the corresponding  $\Sigma$  was found to be “too large to be ignored” (Gille and Davis, 1999). In spite of that, all present day  $z$ -coordinates OGCMs employ (2) with  $\Sigma = 0$ :

$$\bar{\tau}_t + (\bar{\mathbf{U}} + \mathbf{U}^+) \cdot \nabla \bar{\tau} = D(\bar{\tau}) + Q \quad (3)$$

The two most frequently cited reasons to neglect  $\Sigma$  are one of physical nature and one of more mathematical nature. The first one is usually formulated as follows: in an adiabatic regime, the flux of any scalar across isopycnals is zero and thus a diapycnal flux such as  $\Sigma$  should not be present. In Appendix A we show that in an adiabatic regime the *total flux* of any scalar across isopycnals must indeed be zero but we also show that  $\Sigma$  is *not the total buoyancy flux* but only part of it. Therefore, there is no physical justification to require that under adiabatic conditions  $\Sigma$  be zero. The second argument is that the results of a  $z$ -coordinate ocean code using (3) without  $\Sigma$  can be re-interpreted as those derived from the TRM (temporal residual mean) in which there is no  $\Sigma$  (almost by definition since TRM is quite close to an isopycnal model in which there is no  $\Sigma$ ). In Appendix B we present several arguments one of which is that the disappearance of  $\Sigma$  in the TRM occurs at the expense of redefining a *new eddy induced velocity*  $\mathbf{u}^{++}$  which is physically rather different from the standard  $\mathbf{u}^+$  defined above which depends on the mesoscale kinetic energy  $K$ ,  $\mathbf{u}^+(K)$ , Eq. (6c). In fact,  $\mathbf{u}^{++}$  depends on both the eddy kinetic energy  $K$  and the eddy potential energy  $W$ ,  $\mathbf{u}^+(K, W)$ , see Appendix B, Eq. (6). Thus, to model  $\mathbf{u}^{++}$  one needs to model the two mesoscale variables  $K$  and  $W$  which have quite different  $z$ -profiles (Böning and Budich, 1992), a modeling requirement that is no more advantageous than having to model the two variables  $\mathbf{u}^+(K)$  and  $\Sigma$  since we recall that the latter represents the rate of production of eddy potential energy  $W$ . Stated differently, since in a  $z$ -coordinate system one must model two scalars,  $K$  and  $\Sigma$ , and since scalars are unaffected by coordinates transformation, the same two physical parameters must also be present in the TRM formalism in which in fact they appear in the definition of  $\mathbf{u}^{++}$ .

## 2. Model independent results

In this section, we present a series of model independent results preparatory to the modeling procedure discussed in Section 3. To model  $\bar{\mathbf{U}}'\tau'$  in (1), we use the known fact that the physics of mesoscales is more transparent in isopycnal than in  $z$ -coordinates. If we adopt a double prime to denote fluctuating quantities in isopycnal coordinates, it was shown in Canuto and Dubovikov (2006) that the following relations hold true between the fluctuating variables in the two systems of coordinates (see also McDougall and McIntosh (2001), Eq. (39)):

$$\tau'' = \tau' - N^{-2}\bar{\tau}_z b', \quad \mathbf{U}' = \mathbf{U}'' \quad (4a)$$

$$\overline{\mathbf{U}'\tau'} = \mathbf{F}_\tau + \bar{\tau}_z N^{-2} \mathbf{F}_b \quad (4b)$$

where

$$\mathbf{F}_\tau = \overline{\mathbf{U}'\tau''}, \quad \mathbf{F}_b = \overline{\mathbf{U}'b'} \quad (5a)$$

are the tracer and buoyancy fluxes, respectively. If we insert (5a) into (1), the result is different from the “canonical form” (2) and the identification of the different terms is consequently difficult. For that reason, (5a) and (1) must be transformed to acquire a form similar to (2). This process is purely mathematical in nature and has no physical content. For that reason, we present it in Appendix C, Eqs. (7), from which we reproduce the final result:

$$\bar{\tau}_t + (\bar{\mathbf{U}} + \mathbf{U}^+) \cdot \nabla \bar{\tau} + \frac{\partial \Sigma}{\partial z} = D(\bar{\tau}) \quad (5b)$$

where the  $\Sigma$  flux is a combination of tracer and buoyancy fluxes:

$$N^2 \Sigma = \mathbf{F}_\tau \cdot \nabla \bar{b} + \bar{\tau}_z \mathbf{F}_b \cdot \nabla \bar{b} + \overline{\mathbf{u}'b'} \cdot \nabla_\rho \bar{\tau} \equiv N^2(\Sigma_1 + \Sigma_2 + \Sigma_3) \quad (5c)$$

where  $\nabla_\rho = \nabla_H + \mathbf{L} \partial_z$  and where for future convenience we have called the three terms in (5c)  $\Sigma_{1,2,3}$ . At the same time, the diffusion term turns out to be

$$N^{-2} D(\bar{\tau}) = -\nabla_\rho \left[ N^{-2} \left( \overline{\mathbf{u}'\tau'} - \frac{\tau_z}{N^2} \overline{\mathbf{u}'b'} \right) \right] \quad (5d)$$

Finally, the eddy-induced velocity  $\mathbf{U}^+ = (\mathbf{u}^+, w^+)$  is defined by the model independent relations:

$$\mathbf{u}^+ = -\partial_z(N^{-2} \overline{\mathbf{u}'b'}), \quad \nabla_H \cdot \mathbf{u}^+ + \partial_z w^+ = 0 \quad (5e)$$

As a consistency check, consider  $\tau$  to be the buoyancy field,  $\tau = b$ . We obtain:

$$\bar{\tau}_z = N^2, \quad \tau'' = 0, \quad \mathbf{F}_\tau = 0, \quad D(\bar{b}) = 0, \quad \Sigma_{1,3} = 0, \quad \Sigma_2 \equiv \Sigma_b = \mathbf{F}_b \cdot \nabla \bar{b} \quad (5f)$$

in which case Eq. (5b) reduces to the known buoyancy equation (Treguier et al., 1997, Eqs. (6) and (7)). We stress that (5b)–(5e) are model independent. Next, we model the mesoscale functions  $\Sigma$ ,  $D(\bar{\tau})$ .

### 3. Modeling $\Sigma$ and $D(\bar{\tau})$

#### 3.1. The diffusion $D$

The simplest term is the diffusion term (5d) which turns out to be:

$$N^{-2} D(\bar{\tau}) = \nabla_\rho (N^{-2} \kappa_M \nabla_\rho \bar{\tau}) \quad (6a)$$

which, for a constant mesoscale diffusivity  $\kappa_M$ , coincides with Redi's (1982) formula. However, as shown in Canuto and Dubovikov (2006), Eq. (4b), the diffusivity  $\kappa_M$  is not constant.

#### 3.2. The $\Sigma_3$ term in (5c)

Next, we study the  $\Sigma_3$  term in (5c). Using the first of (5e), we have:

$$\mathbf{F}_H = \overline{\mathbf{u}'b'} = -N^2 \int_{-H}^z \mathbf{u}^+(\xi) d\xi \quad (6b)$$

In Canuto and Dubovikov (2006), Eqs. (4), it was shown that the eddy-induced velocity in (6b) has the following form ( $\mathbf{L} = -N^{-2} \nabla_H \bar{b}$ ):

$$\begin{aligned} \mathbf{u}^+ &= -k_M \boldsymbol{\psi}, \quad \kappa_M = r_d K^{1/2}(z) \\ \boldsymbol{\psi} &= \partial_z \mathbf{L} + \boldsymbol{\psi}(\text{new}), \quad \boldsymbol{\psi}(\text{new}) = -A \mathbf{k} \times (\mathbf{u}_d - \bar{\mathbf{u}}) - f^{-1} \boldsymbol{\beta} \end{aligned} \quad (6c)$$

where  $\mathbf{u}_d$  is the eddy drift velocity discussed below and defined as

$$\mathbf{u}_d = \langle \bar{\mathbf{u}} \rangle + (1 + \sigma_t)^{-1} \mathbf{c}_R - A^{-1} \mathbf{k} \times \langle \partial_z \mathbf{L} \rangle \quad (6d)$$

Here,  $A = (1 + \sigma_t^{-1})(fr_d^2)^{-1}$ ,  $\mathbf{c}_R = \sigma_t r_d^2 \mathbf{k} \times \boldsymbol{\beta}$  is the velocity of the barotropic Rossby waves,  $\boldsymbol{\beta} = \nabla_H f$ ,  $\sigma_t$  is the turbulent Prandtl number,  $r_d$  is the Rossby deformation radius, and for an arbitrary function  $f(z)$ , the average  $\langle f \rangle$  is defined as

$$\langle f \rangle \equiv \int_{-H}^0 f(z) K^{1/2}(z) dz \left[ \int_{-H}^0 K^{1/2}(z) dz \right]^{-1} \quad (6e)$$

where  $K(z)$  is the mesoscale kinetic energy given by Eq. (5) in Canuto and Dubovikov (2006). A few comments are necessary. First, the first term in  $\boldsymbol{\psi}$ , Eq. (6c), is the GM model and by itself it does not satisfy the baroclinicity condition:

$$\int_{-H}^0 \mathbf{u}^+(z) dz = 0 \quad (6f)$$

a well-known shortcoming that has been remedied by the use of ad hoc tapering functions which significantly affect the results (Danabasoglu and McWilliams, 1995; Large et al., 1997; Gerdes et al., 1999; Killworth, 2001, 2005). Second, it is simple to show that  $\Psi(\text{new})$  makes (6f) satisfied, that is, we now have (Canuto and Dubovikov, 2006, Eq. (4i)):

$$\int_{-H}^0 \mathbf{u}^+(z) dz = \int_{-H}^0 \kappa_M [\partial_z \mathbf{L} + \Psi(\text{new})] dz = 0 \quad (6g)$$

and thus tapering schemes are no longer necessary.

Third, using an eddy resolving ocean code, Bryan et al. (1999) concluded that the GM model was not sufficient to represent the data and that “additional terms, not related to the gradient of thickness”, were needed. As one can see,  $\Psi(\text{new})$  does not involve the thickness gradient and it may be a candidate of what Bryan et al. described. The term  $\Psi(\text{new})$  is in fact entirely of dynamical nature since it depends on the difference between the mean velocity  $\bar{\mathbf{u}}$  and  $\mathbf{u}_d$  where the latter represents the mesoscales “drift velocity” that characterizes the eddy motion as a whole since “eddies move through the background water at speeds and direction inconsistent with background flow” (Richardson, 1993). Since the drift velocity is the same through the entire vertical eddy’s structure and is thus  $z$ -independent, for the GM model to be valid it is necessary that:

$$\bar{\mathbf{u}}(z) = \mathbf{u}_d \quad (6h)$$

and that  $\beta$  be zero. While the latter may be an acceptable approximation, (6h) is impossible to satisfy everywhere since  $\mathbf{u}_d$  is a  $z$ -independent, barotropic variable while the mean velocity  $\bar{\mathbf{u}}(z)$  is  $z$ -dependent. Though (6h) may be satisfied at some point, it cannot be valid throughout the water column. Since the difference  $\mathbf{u}_d - \bar{\mathbf{u}}$  most likely changes sign along the water column, it may be interpreted as an “erosion” by the mean flow on the otherwise compact, axisymmetric structure represented by an eddy. Energetic considerations that justify more fully this statement were presented in Canuto and Dubovikov (2005) (Eqs. (33a)–(33d)).

In conclusion, combining Eqs. (6b)–(6e) and the model for  $K$  given in Canuto and Dubovikov (2006) (Eqs. (5)), one obtains the final form of the first term  $\mathbf{u}'b'$  in (5c).

### 3.3. The $\Sigma_2$ term in (5c)

Next, the  $\Sigma_2$  term in (5c) was modeled in Canuto and Dubovikov (2006) (Eqs. (7d)–(7h)) with the following results:

$$\mathbf{F}_b \cdot \nabla \bar{b} = -k_m^r N^2, \quad k_m^r = \Gamma_m^r \frac{\varepsilon_m}{N^2} \quad (7a)$$

In order to give a physical interpretation of this term, let us consider the case in which the generic tracer coincides with the buoyancy field in which case Eqs. (5b) and (5f) give:

$$\bar{b}_t + (\bar{\mathbf{U}} + \mathbf{U}^+) \cdot \nabla \bar{b} + \frac{\partial \Sigma_b}{\partial z} = 0, \quad \Sigma_b = \mathbf{F}_b \cdot \nabla \bar{b} \quad (7b)$$

This is the well-known buoyancy equation (Treguier et al., 1997, Eqs. (6) and (7), where our  $\Sigma_b$  is called simply  $\Sigma$ ). While in Canuto and Dubovikov (2006) (Section 4) we presented a detailed discussion of the physical meaning of  $\Sigma_b$ , suffices it to say here that it acts like the source of eddy potential energy  $\text{EPE} = (2N^2)^{-1} \bar{b}^2$ , whose dynamic equation reads:

$$\frac{D}{Dt} \text{EPE} = -\Sigma_b \quad (7c)$$

which further shows that  $\Sigma_b$  must be negative. The closure of  $\Sigma_b$  presented in (7a) reflects this fact and shows that the residual mesoscale diffusivity  $k_m^r$  in (7a) depends on the mesoscale dissipation rate  $\varepsilon_m$  which, once multiplied by the efficiency factor  $\Gamma_m^r$ , represents the total eddy energy production rate. The latter depends on where the EPE is the largest which occurs near 400 m depth. Thus, while we have chosen to cast the model results in a form reminiscent of the Osborn-Cox formulae for the vertical diffusivities, the physics here is quite different since the mesoscale diffusivity  $k_m^r$  is not a constant in  $z$  and neither is constant in geographical locations since mesoscale activities are different in different parts of the ocean. We further recall that using an eddy

resolving code, Gille and Davis (1999) computed  $\Sigma_b$  and found it to be “too large to be ignored”. The functions  $\Sigma_b$ ,  $k_m^r$  and  $\Gamma_m^r$  were plotted in Figs. 4–6 of Canuto and Dubovikov (2006).

### 3.4. The $\Sigma_I$ term in (5c)

Finally, we consider the  $\Sigma_I$  term in (5c). Since it is the most difficult to model and in order to avoid interrupting the presentation, we quote only the final result of the derivation presented in Appendix D. The final expression is

$$\mathbf{F}_\tau \cdot \nabla \bar{b} = \kappa_M (\mathbf{k} \times \mathbf{J}) \cdot \nabla_\rho \bar{\tau}, \quad \mathbf{J} = \frac{1}{2} f (\bar{\mathbf{u}} - \mathbf{u}_d) \partial_z \ln \Gamma \quad (7d)$$

where  $\Gamma(z) \equiv K(z)/K_t$  is the eddy kinetic energy  $K(z)$  normalized to its surface value  $K_t$  which is given in Canuto and Dubovikov (2006) (Eqs. (6)). The physical interpretation of this term is not easy but it is quite clear that it depends entirely on  $\bar{\mathbf{u}}(z) - \mathbf{u}_d$  which we have already discussed after Eq. (6h) and which does not exist in the GM model. This term is clearly related to the additional (dynamical) term that in the eddy induced velocity we have called  $\Psi$ (new) and accounts for the disruptive action of the mean flow on the eddies axisymmetric coherent structure which moves in a flow of different mean velocity.

### 3.5. The complete form of $\Sigma$

Putting together the results for  $\Sigma_{1,2,3}$ , using  $\nabla_\rho = \nabla_H + \mathbf{L} \partial_z$  and rearranging terms, the resulting  $\Sigma$  flux for an arbitrary tracer can be presented as the sum of vertical and horizontal terms as follows:

$$\Sigma = -\mathbf{K}_H \cdot \nabla_H \bar{\tau} - K_\tau \frac{\partial \bar{\tau}}{\partial z} \quad (8a)$$

where the corresponding diffusivities are given by

$$-N^2 \mathbf{K}_H = \overline{\mathbf{u}'b'} + \kappa_M \mathbf{e} \times \mathbf{J}, \quad K_\tau = \kappa_m^r + \mathbf{K}_H \cdot \mathbf{L} \quad (8b)$$

## 4. Magnitude of the new term

First, we notice that both terms in (8b) are of the same order. In fact, from Eqs. (6b) and (5e) it follows that:

$$\overline{\mathbf{u}'b'} \sim HN^2 \mathbf{u}^+ \sim HN^2 k_M \mathbf{L}_z \quad (8c)$$

where  $H \sim 1$  km is the vertical scale. In the definition of  $\mathbf{J}$  in Eq. (7d), we substitute:

$$\partial_z \ln \Gamma \sim H^{-1}, \quad u_d \sim fr_d^2 L_z \quad (8d)$$

where the second relation follows from Eqs. (4e) and (4f) of Canuto and Dubovikov (2006). Since  $NH \sim fr_d$ , we obtain that  $\mathbf{J}$  is of the same order of magnitude as  $\overline{\mathbf{u}'b'}$ . This means that both terms in (8b) are of the same order of magnitude. In addition, from (5e) and (8a) it follows that the new term  $\Sigma_z$  in (5b) is of the same order of magnitude as the  $\mathbf{u}^+$  term.

## 5. Conclusions

The goal of this paper was to derive the dynamic equations for an arbitrary mean tracer to be used in  $z$ -coordinates OGCMs. The correct equation is (2) which contains the additional diapycnal term  $\partial_z \Sigma$  which we have now modeled in Eqs. (8a) and (8b) in terms of the resolved fields. The next step is to use of the new T–S Eq. (2) in a  $z$ -coordinate OGCM to assess the implications of the new  $\Sigma$  term.

## Appendix A. Is a diapycnal flux $\Sigma$ compatible with the adiabatic approximation?

Consider the velocity  $w_s$  across a moving surface (Griffies, 2005):

$$w_s = (\mathbf{U} - \mathbf{U}_s) \cdot \mathbf{n} \quad (1a)$$

where  $\mathbf{U}$  is the fluid velocity,  $\mathbf{U}_s$  is the velocity of the surface and  $\mathbf{n}$  is the unit vector normal to the surface. The flux  $F(\varphi)$  of an arbitrary scalar  $\varphi$  across this surface is given by

$$F(\varphi) = \overline{w_s \varphi} = \overline{(\mathbf{U} - \mathbf{U}_s) \cdot \mathbf{n} \varphi} \quad (1b)$$

where the overbar represents an average over a sub-grid cell and it is meaningful in both isopycnal and  $z$ -coordinates formalisms. From (1b) it follows that the flux is the sum of two fluxes:

$$F(\varphi) = \overline{\mathbf{U} \cdot \mathbf{n} \varphi} - \overline{\mathbf{U}_s \cdot \mathbf{n} \varphi} = \underbrace{F(\varphi)}_{\text{froz.surf.}} + \underbrace{F(\varphi)}_{\text{var.surf.}} \quad (1c)$$

The first term represents the flux across *frozen isopycnal surfaces* while the second represents the flux due to the *variation in time* of the isopycnal surfaces themselves. Clearly, the above relations are valid in any system of coordinates. Next, we apply the above results to isopycnal surfaces which are characterized by the following variables:

$$\mathbf{n} = \mathbf{k} + b_z^{-1} \nabla_H b, \quad \mathbf{U}_s = -\mathbf{k} b_z^{-1} b_t \quad (2a)$$

It follows that:

$$\mathbf{U}_s \cdot \mathbf{n} = -b_z^{-1} b_t, \quad \mathbf{U} \cdot \mathbf{n} = w + b_z^{-1} \mathbf{u} \cdot \nabla_H b \quad (2b)$$

Next, we choose  $\varphi = N^{-2} b_z b'$  ( $N^2 = \bar{b}_z$ ). Substituting the results in (1c), the two fluxes become:

$$\begin{aligned} F_b(\text{froz.surf.}) &= \Sigma + N^{-2} \left( \overline{\mathbf{U} \cdot \nabla \frac{1}{2} b'^2} + \frac{1}{2} \overline{\mathbf{U}' \cdot \nabla b'^2} \right) \\ F_b(\text{var.surf.}) &= N^{-2} \partial_t \frac{1}{2} \overline{b'^2} \end{aligned} \quad (2c)$$

where the diapycnal flux  $\Sigma$  is defined as

$$\Sigma = \overline{w' b'} + N^{-2} \nabla_H \bar{b} \cdot \overline{\mathbf{u}' b'} \quad (2d)$$

Since in an adiabatic regime any flux across isopycnal surfaces must vanish  $F(\varphi) = 0$ , using (1c) and (2c) we obtain:

$$\partial_t \frac{1}{2} \overline{b'^2} + \left( \overline{\mathbf{U} \cdot \nabla \frac{1}{2} b'^2} + \frac{1}{2} \overline{\mathbf{U}' \cdot \nabla b'^2} \right) = -N^2 \Sigma \quad (2e)$$

which is the well-known dynamic equation for the *buoyancy variance* (Treguier et al., 1997). The terms in brackets represent advection and diffusion of the buoyancy variance whereas the right hand side represents its production. McDougall and McIntosh (2001) have argued that the diffusion term is negligible in comparison with advection since the former is a third order term in fluctuating fields and in Dubovikov and Canuto (2005) it was further shown that the advection terms are negligible in comparison with the production. Neglecting such terms, we obtain from Eq. (2c) and (1c):

$$F_b(\text{froz.surf.}) + F_b(\text{var.surf.}) = 0, \quad \Sigma = F(\text{froz.surf.}) \quad (2f)$$

The first relation states the well known fact that *adiabaticity* implies that the total flux across isopycnals is zero. The second relation tells us that  $\Sigma$  is not the total buoyancy flux but only the component corresponding to the frozen isopycnal surfaces. It follows that there is no physical justification to require that  $\Sigma$  must be zero even under adiabatic conditions. However, if for numerical reasons one wants to get rid of  $\Sigma$ , it is possible but not straightforward or unique to choose a gauge transformation to do so (Eden et al., in press).

## Appendix B. Thickness averaged vs. depth-averaged equations

It has been suggested that use of (3) of the text can be justified since it is formally similar to the tracer equation derived within the *thickness-averaged* equations of the TRM formalism (temporal residual mean, McDougall and McIntosh (2001) in which the thickness averaging is represented by a hat, Eq. (37)):

$$\partial_t \hat{\tau} + (\overline{\mathbf{U}} + \mathbf{U}^{++}) \cdot \nabla \hat{\tau} = D(\hat{\tau}) + \mathcal{Q} \quad (1)$$

where  $\mathbf{U}^{++} = (\mathbf{u}^{++}, w^{++})$  is the TRM eddy-induced velocity. Eq. (1) is Eq. (53) of McDougall and McIntosh (2001), in which the symmetric part  $S$  was written as a diffusion  $D$ ; note that McDougall and McIntosh (2001) call  $\mathbf{U}^+$  what we call  $\mathbf{U}^{++}$ . The similarity between (1) above and (3) of the text has led to the view that the solutions of the latter are equivalent to those of (1) above provided one re-interprets  $\bar{\tau}$  as  $\hat{\tau}$ . If that were the case, there would be no need to model  $\Sigma$ . However, Eqs. (1) and (3) exhibit important differences that we analyze in detail: first, we need to recall that in the TRM formalism,  $\mathbf{U}^{++} = (\mathbf{u}^{++}, w^{++})$  is related to the Eulerian eddy induced velocity  $\mathbf{U}^+$  in Eq. (3) by the following relations obtained from Eqs. (19) and (4b) of McDougall and McIntosh (2001):

$$\mathbf{u}^{++} = \mathbf{u}^+ + \frac{\partial}{\partial z}(N^{-2}W\bar{\mathbf{u}}_z), \quad W = \frac{1}{2}N^2\rho_z^{-2}\overline{\rho'^2} = \frac{1}{2}N^2\overline{b'^2} \quad (2)$$

where  $W$  is the eddy potential energy (as suggested by MMI, in (2) we have neglected a cubic term in perturbation amplitude). The question now is ‘Are Eqs. (3) of the text and (1,2) above the same?’ To answer the question, we analyze below several aspects of the problem.

(1) Are  $\mathbf{U}^{++}$  and  $\mathbf{U}^+$  in (1) and (3) the same?

Using the geostrophic relation for  $\bar{\mathbf{u}}_z$  and the GM model for the eddy induced velocity:

$$\frac{\partial \bar{\mathbf{u}}}{\partial z} = -f^{-1}N^2\mathbf{k} \times \mathbf{L}, \quad \mathbf{u}^+ \sim \partial_z(\kappa_M \mathbf{L}) \quad (3)$$

where  $\kappa_M \sim 10^3 \text{ m}^2 \text{ s}^{-1}$  is the mesoscale diffusivity and  $\mathbf{L} = -N^{-2}\nabla_H \bar{b}$  is the slope of the isopycnals, the ratio of  $\mathbf{u}^+$  to the next term in the first relation in (2) turns out to be:

$$\frac{\partial_z(\kappa_M \mathbf{L})}{\partial_z(f^{-1}W\mathbf{k} \times \mathbf{L})} \sim \kappa_M f W^{-1} \sim O(1) \quad (4)$$

where for  $W$  we have used the results of Böning and Budich (1992, Fig. 6) as well as Fig. 3 of Canuto and Dubovikov (2006) and where the mesoscale diffusivity scales like (Canuto and Dubovikov, 2005, 2006, Eq. (4b))

$$\kappa_M \sim r_d K^{1/2}(z) \quad (5)$$

Here,  $r_d$  is the Rossby deformation radius and  $K(z)$  is the mesoscale kinetic energy. Therefore, the two terms in  $\mathbf{u}^{++}$  are of the same order of magnitude. What is the physical difference between  $\mathbf{u}^+$  and  $\mathbf{u}^{++}$ ? Because of (3a), (4) and (5),  $\mathbf{u}^+$  depends on  $K$  whereas  $\mathbf{u}^{++}$  depends also on the eddy potential energy  $W$ , so that we can write:

$$\mathbf{u}^+(K), \quad \mathbf{u}^{++}(K, W) \quad (6)$$

The mesoscale functions  $K$  and  $W$  have different  $z$ -profiles and peak at different depths. Specifically,  $K(z)$  has its maximum at  $z = 0$  (surface) whereas  $W$  has its minimum at the surface and its maximum at  $\sim 500$  m, as seen from Fig. 6 of Böning and Budich (1992), who have noted: while  $K$  decreases with increasing depth through the main thermocline, the maximum values of  $W$  occurs at the subsurface.

There is an additional feature that differentiate  $\mathbf{u}^{++}$  from  $\mathbf{u}^+$ . Because of (3) above the second term in (2) depends on  $f$  which has different signs in the northern and southern hemisphere whereas  $\mathbf{u}^+$  does not do so.

(2)  $\mathbf{U}^{++}$  and  $\Sigma$ .

To better understand the physical contribution of  $\mathbf{U}^{++}$  and  $\Sigma$  to the mean tracer equation, we consider (3a) in the case of buoyancy. The second term in (2) contributes the term:

$$\Gamma = \frac{\partial}{\partial z}(N^{-2}W\bar{\mathbf{u}}_z) \cdot \nabla_H \bar{b} = -Wf^{-1}(\mathbf{k} \times \nabla_H N^2) \cdot \mathbf{L} \quad (7a)$$

Next, for an adiabatic ocean, we employ the dynamic equation for the potential energy in which we neglect the advective terms (Canuto and Dubovikov, 2006, Eq. (14g); Treguier et al., 1997, Eq. (12)):

$$D_t W = -\Sigma, \quad \Sigma = -t_{\text{mes}}^{-1} W \quad (7b)$$

where  $t_{\text{mes}}^{-1}$  is the growth rate of  $W$ . We obtain:

$$\Gamma = \ell^{-1} \Sigma, \quad \ell^{-1} = f^{-1} t_{\text{mes}} \nabla_H N^2 \times \mathbf{k} \cdot \mathbf{L} \sim 2 \cdot 10^3 L_h^{-1} \sim 10^{-3} \text{ m}^{-1} \quad (7c)$$



where we have used  $t_{\text{mes}} \sim \sim$  a few months,  $f \sim 10^{-4} \text{ s}^{-1}$ ,  $N^2 \sim 10^{-5} \text{ s}^{-2}$ , an isopycnal slope of  $10^{-3}$  and  $L_h$  is a typical horizontal scale. The above derivation shows that from the physical viewpoint, the second term in the definition of  $\mathbf{u}^{++}$  in (2) contributes to (1) a term  $\Gamma$  that is of the same order as the  $\partial \Sigma / \partial z \sim \Sigma \ell^{-1}$  term in Eq. (2) of the text.

In summary, the two terms in  $\mathbf{u}^{++}$  depend on two scalar functions  $K$  and  $W$  that represent different physical properties of mesoscales, have different  $z$ -profiles and a model for one of them cannot be expected to work for the other as well. If a formalism (like the Eulerian one) requires to model two mesoscales variables, *eddy kinetic and potential energies*, the same must be true in any other system of coordinates. Since  $W$  is a scalar and thus independent of any system of coordinates, it cannot be transformed away. From the physical viewpoint, since the eddy potential energy  $W$  is the “recipient” of the energy from the (mean field) baroclinic instabilities,  $W$  represents a key physical variable of the system no matter how one exhibits it. Thus, it is logical to expect that  $W$ , or a physically equivalent of it like  $\Sigma$ , should appear explicitly in the problem. In that sense, the disappearance of one of them as in (1) above is purely formal since it would be quite hard to justify depriving the basic equations of a key feature of mesoscale physics such as  $W$ . The practical implication is that a model for  $\mathbf{u}^+$  cannot be assumed to be valid for  $\mathbf{u}^{++}$  as well. We have explicitly tested this assertion in Canuto and Dubovikov (2006): the modeling of  $\mathbf{u}^+$  and  $W$  yielded different results (see Eqs. (4) and (7a)) which were checked against numerical simulations (Böning and Budich, 1992) and heuristic models (Karsten and Marshall, 2002; Olbers and Visbeck, in press). In conclusion, whether one chooses  $(\mathbf{U}^+, \Sigma)$  or  $\mathbf{U}^{++}$ , there are always two variables to model.

(3) Are the two  $\bar{\mathbf{U}}$  in (1) and (2) the same?

To answer the question, we note that in Eqs. (2)  $\bar{\mathbf{U}}$  is the solution of the OGCMs depth-averaged mean momentum equation ( $R_{ij}$  represent the Reynolds stresses and we take  $\rho_0 = 1$ ):

$$\partial_t \bar{\mathbf{u}} + \bar{\mathbf{U}} \cdot \nabla \bar{\mathbf{u}} - f \bar{v} = -\bar{p}_x - \nabla \cdot \mathbf{R}|_x \quad (8a)$$

while the  $\bar{\mathbf{U}}$  entering (1) above is the solution of the thickness-averaged mean momentum equations given by Eq. (66) of McDougall and McIntosh (2001) which, to make the comparison more transparent, we rewrite as

$$\partial_t (\bar{\mathbf{u}} + \mathbf{u}^{++}) + (\bar{\mathbf{U}} + \mathbf{U}^{++}) \cdot \nabla (\bar{\mathbf{u}} + \mathbf{u}^{++}) - f \bar{v} = -\bar{p}_x - \nabla \cdot \mathbf{R}|_x \quad (8b)$$

To highlight the comparison with (8a), we rewrite (8b) as follows:

$$\partial_t \bar{\mathbf{u}} + \bar{\mathbf{U}} \cdot \nabla \bar{\mathbf{u}} - f \bar{v} = -\bar{p}_x - \nabla \cdot \mathbf{R}|_x - \mathbf{Rest} \quad (8c)$$

where:

$$\mathbf{Rest} \equiv \partial_t \mathbf{u}^{++} + (\bar{\mathbf{U}} + \mathbf{U}^{++}) \cdot \nabla \mathbf{u}^{++} + \mathbf{U}^{++} \cdot \nabla \bar{\mathbf{u}} \quad (8d)$$

which we now study. Since the largest term in (8d) is the  $z$ -component of the last term  $w^{++} \partial_z \bar{u}$ , we must compare it with other terms in (8c) to assess its relevance. We begin by comparing it with the horizontal component of the advection term  $\bar{\mathbf{U}} \cdot \nabla \bar{\mathbf{u}}$  in (8c). Using the first relation in (3a) above, we derive that:

$$\frac{w^{++} \partial_z \bar{u}}{\bar{u} \partial_x \bar{u}} \sim \frac{w^{++} N^2 |\mathbf{L}|}{\bar{u}^2 f L_h^{-1}} \sim \frac{w^{++}}{\bar{u}} \frac{H N^2}{f \bar{u}} \sim O(1) \quad (8e)$$

where we have used:

$$w^{++} \sim u^{++} H / L_h, \quad \partial_x \sim L_h^{-1}, \quad f \sim 10^{-4} \text{ s}^{-1}, \quad N^2 \sim 10^{-5} \text{ s}^{-2} \quad (8f)$$

$$|\mathbf{L}| \sim H / L_h \sim 10^{-3}, \quad u^{++} \sim 0.1 \bar{u}, \quad h \sim 10^3 \text{ m} \quad (8g)$$

Here,  $H$  and  $L_h$  are the vertical and horizontal scales. It follows that the largest of the **Rest** terms in (8d) is not negligible when compared with the other terms in (8c).

Next, we compare  $w^{++} \partial_z \bar{u}$  to the vertical component of the advective term in (8c). We have:

$$\frac{w^{++} \partial_z \bar{u}}{\bar{w} \partial_z \bar{u}} = \frac{w^{++}}{\bar{w}} \sim \frac{0.1 f}{\beta L_h} \sim O(1) \quad (8h)$$

where we used the relations:

$$\bar{w} \sim f^{-1} \beta h \bar{u}, \quad \beta = \partial_y f, \quad \bar{u} \sim \bar{v} \quad (8i)$$



In deriving the first relation in (8i), we employed the geostrophic relation (3a) above plus the continuity equation. The two relation (8e) and (8h) show that the **Rest** terms in (4c) are of the same order (or larger) than the other terms in (4c) thus making (8c) and (8a) different equations that yield different  $\bar{\mathbf{U}}$ 's. Therefore, for a given  $\bar{\tau}$ , the two  $\bar{\mathbf{U}}$  in Eqs. (2) of the text and (1) above are not the same function.

(4) Mixed layer.

In the ML neither an isopycnal nor the TRM formalism are applicable and one must resort to  $z$ -coordinates. From the practical viewpoint, matching a TRM code in the adiabatic region with a  $z$ -coordinates code in the ML may not be easy to do because of the different physical meaning of the variables  $\bar{\tau}$  and  $\hat{\tau}$  whereas use of  $z$ -coordinates greatly facilitates the matching problem.

### Appendix C. Model independent results

We follow the standard procedure of decomposing fluxes in *diapycnal* and *isopycnal* components. For the flux  $\mathbf{F}_\tau$ , Eq. (5a), we have:

$$\mathbf{F}_\tau = \underbrace{\mathbf{k}N^{-2}\mathbf{F}_\tau \cdot \nabla \bar{b}}_{\text{diapycnal}} + \underbrace{\bar{\mathbf{u}}'\tau'' + \mathbf{k}(\bar{\mathbf{u}}'\tau'' \cdot \mathbf{L})}_{\text{isopycnal}} \quad (1)$$

where  $\mathbf{u}'$  is the horizontal velocity,  $\mathbf{L} = -N^{-2}\nabla_H \bar{b}$  is the slope of the isopycnals and  $\mathbf{k}$  is the unit vector in the  $z$ -direction. Applying the operator  $\nabla \cdot$  to (1), we obtain ( $\nabla_\rho = \nabla_H + \mathbf{L}\partial_z$ )

$$\nabla \cdot \mathbf{F}_\tau(\text{dia}) = \frac{\partial}{\partial z}(N^{-2}\mathbf{F}_\tau \cdot \nabla \bar{b}) \equiv \frac{\partial \Sigma_1}{\partial z} \quad (2)$$

$$\nabla \cdot \mathbf{F}_\tau(\text{is}) = N^2 \nabla_\rho(N^{-2}\bar{\mathbf{u}}'\tau'') \equiv -D(\bar{\tau}) \quad (3)$$

Eq. (2) represents the divergence of a diapycnal flux  $\mathbf{k}\Sigma_1$  while (3) represents diffusion. As a first intermediate result, the divergence of (1) is given by

$$\nabla \cdot \bar{\mathbf{U}}'\tau' = -D(\bar{\tau}) + \frac{\partial \Sigma_1}{\partial z} + \nabla \cdot \bar{\tau}_z N^{-2}\mathbf{F}_b \quad (4)$$

This result shows that the decomposition (1) has naturally lead to the appearance of the diffusion term that appears in (2a). Clearly, the last two terms in the rhs of (4) must represent the other mesoscale terms in (2a), namely the eddy-induced velocity and the  $\Sigma$ -term. Let us begin with the last term in (4). First, we decompose  $\mathbf{F}_b$  in the same way as we have decomposed (1) and then take the divergence. In so doing, we obtain relations analogous to (2) and (3) which are as follows:

$$\nabla \cdot \underbrace{\bar{\tau}_z N^{-2}\mathbf{F}_b}_{\text{diapycnal}} = \frac{\partial}{\partial z}(\bar{\tau}_z N^{-2}\mathbf{F}_b \cdot \nabla \bar{b}) \equiv \frac{\partial \Sigma_2}{\partial z} \quad (5a)$$

$$\nabla \cdot \underbrace{\bar{\tau}_z N^{-2}\mathbf{F}_b}_{\text{isopycnal}} = \bar{\tau}_z N^{-2}\mathbf{U}^+ \cdot \nabla \bar{b} + \bar{\mathbf{u}}'b' \cdot \nabla_\rho(\bar{\tau}_z N^{-2}) \quad (5b)$$

In deriving (5a), we used the well-known model independent relations (Canuto and Dubovikov, 2005, 2006)

$$\mathbf{u}^+ = -\partial_z(N^{-2}\bar{\mathbf{u}}'b'), \quad \nabla \cdot \mathbf{U}^+ = 0, \quad \nabla \cdot \mathbf{F}_b(\text{is}) = \mathbf{U}^+ \cdot \nabla \bar{b} \quad (5c)$$

Inserting (5a) and (5b) into (4) we obtain

$$\nabla \cdot \bar{\mathbf{U}}'\tau' = -D(\bar{\tau}) + \bar{\tau}_z N^{-2}\mathbf{U}^+ \cdot \nabla \bar{b} + \bar{\mathbf{u}}'b' \cdot \nabla_\rho(\bar{\tau}_z N^{-2}) + \frac{\partial(\Sigma_1 + \Sigma_2)}{\partial z} \quad (5d)$$

If we now substitute (5d) into (1) of the main text we obtain

$$\bar{\tau}_t + \bar{\mathbf{U}} \cdot \nabla \bar{\tau} + \bar{\tau}_z N^{-2}\mathbf{U}^+ \cdot \nabla \bar{b} + \bar{\mathbf{u}}'b' \cdot \nabla_\rho(\bar{\tau}_z N^{-2}) = D(\bar{\tau}) - \partial_z(\Sigma_1 + \Sigma_2) + \mathcal{Q} \quad (6a)$$

This equation is mathematically correct but hard to interpret physically. To make it more transparent, we add to both sides of (6a) the term  $-\partial_z \Sigma_3$  where

$$\Sigma_3 = N^{-2}\bar{\mathbf{u}}'b' \cdot \nabla_\rho \bar{\tau} \quad (6b)$$

and employ the easy to prove relations

$$\partial_z \Sigma_3 = \overline{\mathbf{u}'b'} \cdot \nabla_\rho (\bar{\tau}_z N^{-2}) - \mathbf{u}^+ \cdot \nabla_\rho \bar{\tau} \quad (6c)$$

$$\mathbf{U}^+ \cdot \nabla \bar{\tau} = \bar{\tau}_z N^{-2} \mathbf{U}^+ \cdot \nabla \bar{b} + \mathbf{u}^+ \cdot \nabla_\rho \bar{\tau} \quad (6d)$$

The tracer Eq. (6a) simplifies considerably since it acquires the final model independent form

$$\bar{\tau}_t + (\bar{\mathbf{U}} + \mathbf{U}^+) \cdot \nabla \bar{\tau} + \frac{\partial \Sigma}{\partial z} = D(\bar{\tau}) \quad (7a)$$

where

$$N^2 \Sigma = \overline{\mathbf{u}'b'} \cdot \nabla_\rho \bar{\tau} + \bar{\tau}_z F_b \cdot \nabla \bar{b} + \mathbf{F}_\tau \cdot \nabla \bar{b} \quad (7b)$$

$$\mathbf{F}_\tau = \overline{\mathbf{U}'\tau''}, \quad \mathbf{F}_b = \overline{\mathbf{U}'b'}, \quad \tau'' = \tau' - N^{-2} \bar{\tau}_z b' \quad (7c)$$

#### Appendix D. The $\Sigma_1$ term in Eq. (5c)

To the main order in the small parameter  $h''/\bar{h}$ , we have  $\mathbf{U}' = \mathbf{U}''$  and  $\nabla_H \bar{b} = -N^2 \nabla_\rho \bar{z}$ , where  $z$  is the height of isopycnal surface. Then, from (4a), we obtain  $\Sigma_1 = \overline{\tau''(w'' - \mathbf{u}'' \cdot \nabla_\rho \bar{z})}$ . In the adiabatic approximation, we have  $w = z_t + \mathbf{u} \cdot \nabla_\rho z$  whose fluctuating component is given by  $w'' = z_t'' + \mathbf{u}'' \cdot \nabla_\rho \bar{z} + \bar{\mathbf{u}} \cdot \nabla_\rho z'' + \mathbf{u}'' \cdot \nabla_\rho z''$ . Substituting  $w'$  in the previous expression and neglecting the third-order terms in the fluctuating fields, we obtain  $\Sigma_1 = \overline{\tau''(z_t'' + \bar{\mathbf{u}} \cdot \nabla_\rho z'')}$ . If we Fourier transform in both time and space, the expression in parenthesis becomes  $(-i\omega + i\bar{\mathbf{u}} \cdot \mathbf{q})z'$  where  $\mathbf{q}$  is a 2D wave vector. In Canuto and Dubovikov (2006) and Dubovikov and Canuto (2005), we showed that for mesoscale eddies  $\omega = \mathbf{q} \cdot \mathbf{u}_d$ , where  $\mathbf{u}_d$  is the eddy drift velocity whose expression in terms of large scale fields is given in the references just cited. Thus, the Fourier transform can be written as  $i(\bar{\mathbf{u}} - \mathbf{u}_d) \cdot \mathbf{q}z'$  which allows us to write  $\Sigma_1 = (\bar{\mathbf{u}} - \mathbf{u}_d) \cdot \overline{\tau'' \nabla_\rho z'}$ . Taking the mesoscale field as quasi-geostrophic, we have that in isopycnal coordinates  $\nabla_\rho z' = f\mathbf{k} \times \partial_b \mathbf{u}''$ . We also notice that the mesoscale fields  $\mathbf{u}''$  and  $\partial_b \mathbf{u}''$  have the same (or opposite) direction since  $\mathbf{u}''$  represents the circular motion of the eddies around their center. Thus,  $\partial_b \mathbf{u}' = (\mathbf{u}'/u')\partial_b u' = \frac{1}{2}\mathbf{u}'\partial_b \ln \Gamma = (2N^2)^{-1}\mathbf{u}'\partial_z \ln \Gamma$ , where  $\Gamma(z) \equiv K(z)/K_s$  is the eddy kinetic energy normalized to the surface eddy kinetic energy  $K_s$ ; in Canuto and Dubovikov (2006) both  $K(z)$  and  $K_s$  were expressed in terms of large scale fields. The final expression for  $\Sigma_1$  then becomes Eq. (7d).

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